# Assignment 11

Coverage: 16.7 in Text.

Exercises: 16.7 no 3, 6, 8, 15, 17, 25.

Hand in 16.7 no 6, 15, 25 by April 20.

**Supplementary Problems**

1. Let $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$. Establish Lagrange identity

$$\sum_{1 \le i < j \le n} (a_i b_j - a_j b_i)^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2 \ .$$

2. Deduce from (1) the identity

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta \ ,$$

   where $\theta$ is the angle between 3-vectors $\mathbf{a}$ and $\mathbf{b}$.

3. A regular parametrization $\mathbf{r}$ from the square $[0,1]^2$ to $S \subset \mathbb{R}^3$ is called a tube if (a) it is bijective on $(0,1)^2$ and (b) $\mathbf{r}((0,y)) = \mathbf{r}((1,y)),\ y \in [0,1]$. Show that for any irrotational $C^1$-vector field $\mathbf{F}$,

$$\oint_{C_1} \mathbf{F} \cdot d\mathbf{r} = \oint_{C_2} \mathbf{F} \cdot d\mathbf{r} \ ,$$

   where $C_1 : x \mapsto \mathbf{r}(x,0)$ and $C_2 : x \mapsto \mathbf{r}(x,1)$ for $x \in [0,1]$.

4. (Optional) Let $\mathbf{r} : D \to S$ be a regular parametrization of $S$ so that $\mathbf{r}_u \times \mathbf{r}_v$ is the chosen normal direction of $S$. Let $\gamma(t)$ be a parametrization of the boundary of $D$ in anticlockwise direction. Show that the curve $\mathbf{r}(\gamma(t))$ described the boundary of $S$ in the orientation induced by the chosen normal of $S$.